TONBRIDGE SCHOOL

Scholarship Examination 2004

MATHEMATICS I

Tuesday 4th May 2004
9.00 a.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. Solve the simultaneous equations below, and give a check that your answer is correct:

$$2y - 0.1x = 8$$
$$3x - 0.4y = 58$$

[8 marks]

2. A sequence of numbers is defined by the rules:

FIRST NUMBER = 3

$$\text{NEXT NUMBER} = \frac{-1}{1 + \text{PRESENT NUMBER}}$$

(a) Show that the 2nd number is $-\frac{1}{4}$ and find the 3rd, 4th, 5th, 6th numbers.

(b) Use your answers to predict both the 450th number in the sequence and the sum of the first 450 numbers. [8]

3. In appropriate units, the rate, V, at which water under pressure P flows through a tube of radius R and length L is given by the formula $V = \frac{400PR^4}{L}$.

(a) If $R = 0.03$, $P = 600$, $L = 0.2$, find V.

(b) If $L = 0.5$, $V = 1.3$, $R = 0.04$, find P.

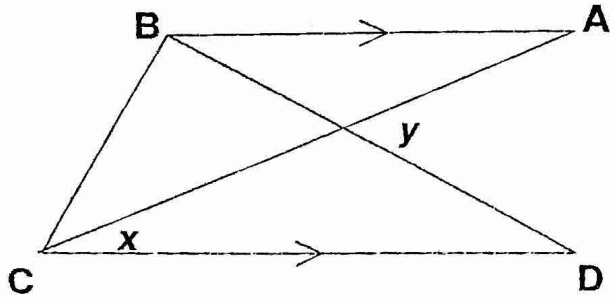
(c) If $L = 0.7$, $V = 0.8$, $P = 220$, find R. [8]

4. The number 325 is equal to a square number multiplied by a prime number.

(a) What are these two numbers?

(b) What is the next number above 325 that consists of a square number multiplied by a prime number? Explain your reasoning carefully. [8]

5. In the diagram below, AB and CD are parallel lines. Also, CA bisects angle BCD and BD bisects angle ABC.



(a) If $x = 20°$, find y.

(c) In general, what is the connection between x and y? Justify your answer. [8]

6. For each of the following containers, find the length of the longest straight stick that will fit into the container without poking out. (You should take all dimensions of the stick other than its length to be negligible.)

(a) A cylinder with radius 10 cm and height 20 cm.

(b) A symmetrical cone with base radius 10 cm and perpendicular height 20 cm.

(c) A symmetrical cone with base radius 10 cm and perpendicular height 15 cm.

(d) A cube with side-length 10 cm.

[15]

7. A curve has the equation $y^2 = x(x + 4)$.

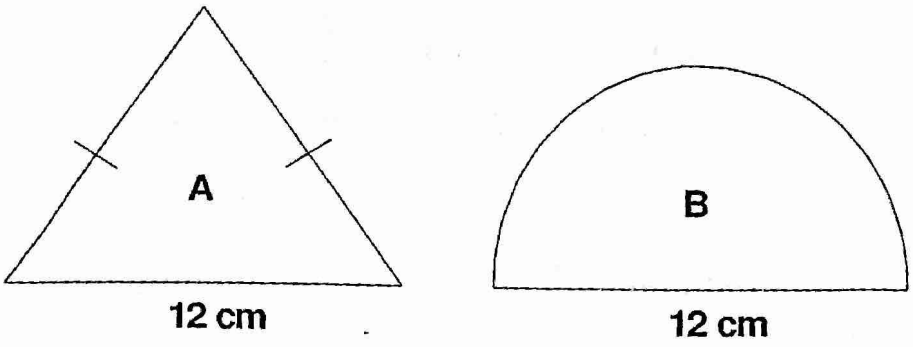
(a) Calculate the (positive) values of y corresponding to $x = 0, 0.5, 1, 1.5, 2, 3, 4, 5$.

(b) Using a scale of 2 cm to 1 unit on the x and y axes, draw a graph of y against x. Comment on the shape of the curve.

(c) A square has sides parallel to the axes and one diagonal joining (5,0) to a point on the curve. What is the area of this square?

[15]

8. The diagram below shows an isosceles triangle (A) and a semicircle (B). The triangle has a base of length 12 cm and the semicircle has diameter 12 cm.



(a) Find the area of A if A and B have the same perimeter.

(b) Find the perimeter of A if A and B have the same area.

[15]

9. A *magic square* is a square array of numbers with the property that all rows, columns and both main diagonals have the same total.
Find the numbers A, B, C, D in the magic square below (which was depicted in an engraving by the artist Dürer).

16	3	2	13
5	A	B	8
9	C	D	12
4	15	14	1

[15]

Mathematics I 2004

1) $2y - 0.1x = 8 \Rightarrow -0.1x + 2y = 8 \Rightarrow -x + 20y = 80$

$3x - 0.4y = 58 \quad 3x - 0.4y = 58 \quad 30x - 4y = 580 \quad \times 5$

$-x + 20y = 80$

$150x - 20y = 2900$

$149x = 2980$

$x = 20$

$\Rightarrow -20 + 20y = 80$

$20y = 100$

$y = 5$

$x = 20, y = 5$

check: $2(5) - 0.1(20) = 10 - 2 = 8$ ✓

$3(20) - 0.4(5) = 60 - 2 = 58$ ✓

2) 1st number $= 3 = \frac{3}{1}$

a) 2nd number $= \frac{-1}{1+3} = -\frac{1}{4}$ $\quad c^{th} = -\frac{4}{3}$

3rd number $= \frac{-1}{1+(-\frac{1}{4})} = \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$

4th number $= \frac{-1}{1+(-\frac{4}{3})} = \frac{-1}{-\frac{1}{3}} = \frac{3}{1}$

5th number $= \frac{-1}{1+3} = -\frac{1}{4}$

b) $3, -\frac{1}{4}, -\frac{4}{3}, 3, -\frac{1}{4}, -\frac{4}{3}, 3$

Repeats every 3 terms

$\frac{450}{3} = 150$ therefore 450th term is $-\frac{4}{3}$

$150\left(3 - \frac{1}{4} - \frac{4}{3}\right) = 150\left(\frac{17}{12}\right) = 212.5$

3) $V = \frac{400PR^4}{L}$

a) $V = \frac{400(600)(0.03)^4}{0.2}$

$V = 0.972$

b) $1.3 = \frac{400P(0.04)^4}{0.5}$

$0.65 = 0.001024P$

$P = 634.766$

c) $0.8 = \frac{400(220)R^4}{0.7}$

$0.56 = 88000R^4$

$R^4 = \frac{0.56}{88000}$

$R = \left(\frac{0.56}{88000}\right)^{\frac{1}{4}}$

$R = 0.05023$

4) 325

11

5 · 65

11

5 · 13

a) $325 = 5^2 \times 13$

numbers = 25 and 13

b) $326 = 2 \times 163$

$327 = 3 \times 109$

$328 = 8 \times 41$

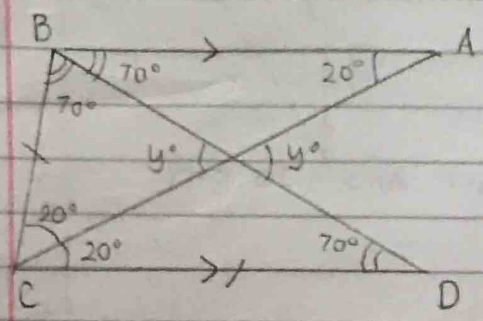
$329 = 7 \times 47$

$330 = 3 \times 110$

$331 = 1 \times 331$ 1 is square so 1×331 also

$332 = 4 \times 83$

↓ square ↓ prime

5) 

a) $x = 20°$

$\angle CBD$ & $\angle CDB = \frac{180-40}{2} = 70°$ $\quad \angle y = 90°$

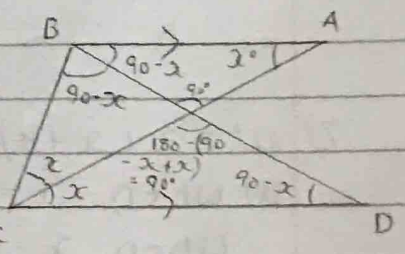
or $\angle ABC = 180 - 40 = 140 \Rightarrow \angle ABD$ & $\angle DBC = 70°$

b) let $x = x°$

$\angle BCA = x°$

$\angle ABC = 180 - 2x$

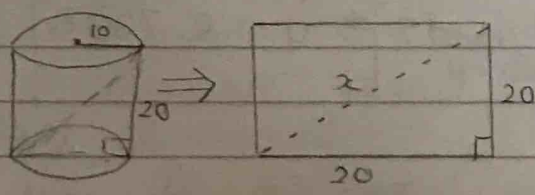
$\angle ABD$ & $\angle DBC = \frac{180-2x}{2} = 90 - x$



$\angle BDC = 180 - (90 - x) - (2x) = 90 - x$ also since alternate int.

No matter what the value of x is, y is always $90°$

c)

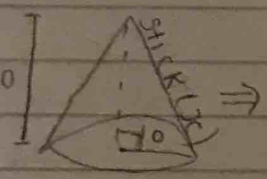
a) 

The longest distance in a cylinder is the hypotenuse of a right triangle with diameter on one side and height on the other

$20^2 + 20^2 = x^2$

$x^2 = 800$

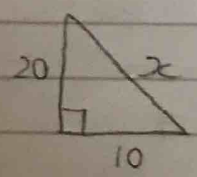
$x = 28.28$

b) 

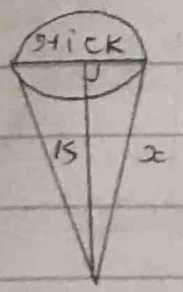
$x^2 = 10^2 + 20^2$

$x^2 = 500$

$x = 22.36$



c)

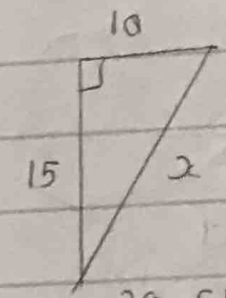


$x^2 = 10^2 + 15^2$

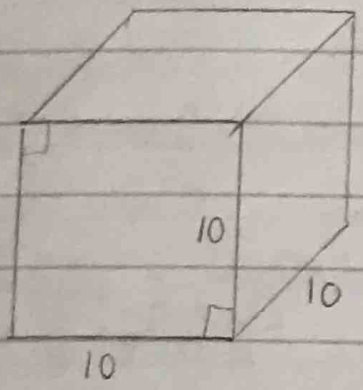
$x^2 = 325$

$x = 18.03$

20 cm stick longer Ans = 20 cm



d)



$x = \sqrt{10^2 + 10^2 + 10^2} = \sqrt{300} = 17.32$

7) $y^2 = x(x+4)$

a) When $x = 0$: $y^2 = 0(4) = 0 \Rightarrow y = 0$

When $x = 0.5$: $y^2 = 0.5(4.5) = 2.25 \Rightarrow y = 1.5$

When $x = 1$: $y^2 = 1(5) = 5 \Rightarrow y = 2.236$

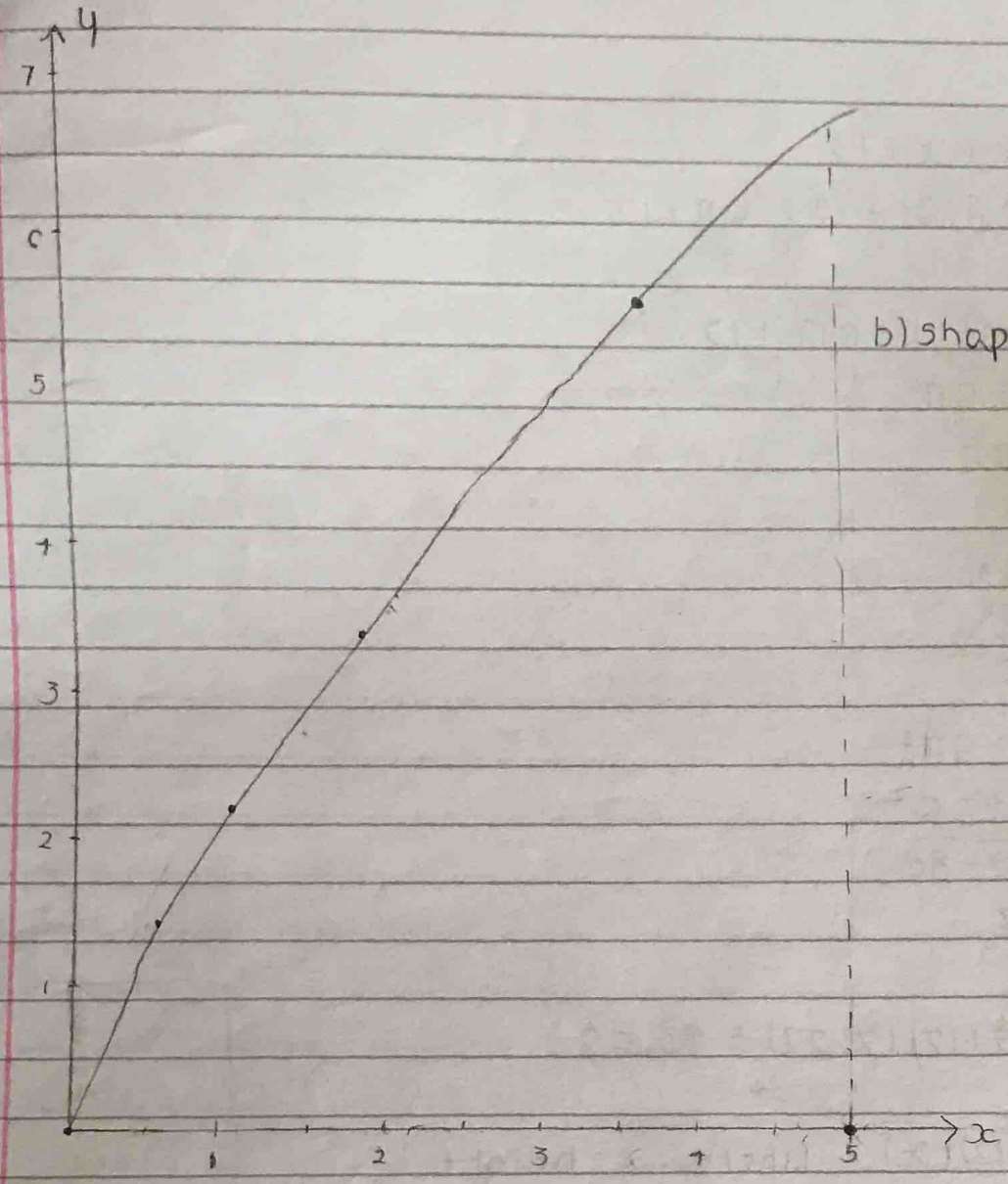
When $x = 1.5$: $y^2 = 1.5(5.5) = 8.25 \Rightarrow y = 2.872$

When $x = 2$: $y^2 = 2(6) = 12 \Rightarrow y = 3.464$

When $x = 3$: $y^2 = 3(7) = 21 \Rightarrow y = 4.583$

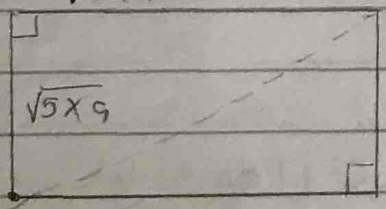
When $x = 4$: $y^2 = 4(8) = 32 \Rightarrow y = 5.657$

When $x = 5$: $y^2 = 5(9) = 45 \Rightarrow y = 6.708$



b) shape - very steep curve
- not a straight line

c)



$\sqrt{5(5+4)}$

When $x = 5$

$y = \sqrt{5(9)} = \sqrt{45}$

$y = \sqrt{x(x+4)} = \sqrt{x^2+4x}$

Area $= \sqrt{45} \times \sqrt{45} = 45$

8)

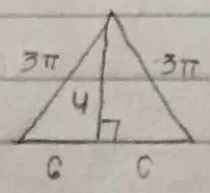
a) A: $x + x + 12$

B: $\frac{2\pi(6) + 12}{2} = 6\pi + 12$

$\Rightarrow x + x + 12 = 6\pi + 12$

$2x = 6\pi$

$x = 3\pi$



$6^2 + y^2 = 9\pi^2$

$y^2 = 9\pi^2 - 6^2$

$y = \sqrt{9\pi^2 - 36}$

$y = 7.27$

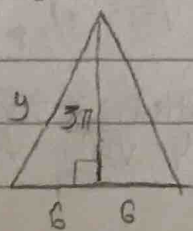
Area $= \frac{1}{2}(12)(7.27) = 43.62$

b) A: $\frac{1}{2}(12)(x)$ where x = height

B: $\pi r^2 = \pi(6)^2 = \frac{36\pi}{2} = 18\pi$

$6x = 18\pi$

$x = 3\pi$



$6^2 + (3\pi)^2 = y^2$

$y = \sqrt{36 + 9\pi^2} = 11.173$

Perimeter of A: $12 + 11.173 + 11.173 = 34.346$

9) $3 + A + C + 15 = 34 \Rightarrow A + C = 16$ & since $C + D = 13 \Rightarrow A + (13 - D) = 16$

$A - D = 3$

$2 + B + D + 14 = 34 \quad B + D = 18$

$5 + A + B + 8 = 34 \quad A + B = 21$

$9 + C + D + 12 = 34 \quad C + D = 13$

$16 + A + D + 1 = 34 \quad A + D = 17$

$4 + C + B + 13 = 34 \quad C + B = 17$

$A + D = 17$

$A - D = 3$

$2A = 20$

$A = 10, D = 7$

Since $B + D = 18 \Rightarrow B = 18 - 7 = 11$

$A = 10$

$B = 11$

$D = 7$

$C + D = 13$

$C = 13 - 7 = 6$

$A = 10, B = 11, C = 6, D = 7$